Math 5B Test 3
SAMPLE
(11.8-11.11, 8.1-8.3)

100 POINTS **NAME:** _____

No scratch paper. Show all work clearly on test paper. No credit will be given for solutions if work is not shown. Only non-graphing calculators are allowed. Unless otherwise specified, the answer to series questions should be given using sigma notation. Unless otherwise stated, you do not need to find the radius of convergence.

(1) FIND THE INTERVAL OF CONVERGENCE FOR EACH OF THE FOLLOWING. (5 points each)

(a) $\sum_{n=1}^{\infty} \frac{2^n (x+3)^n}{\sqrt{n}}$

(b) $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \bullet 4 \bullet 6 \bullet \cdots \bullet (2n)}$

(c) $\sum_{n=1}^{\infty} \frac{n!}{3^n} (x+4)^n$

(2) Find the centroid of the region bounded by the graphs of $y = e^{3x}$, y=0, x=0, and x=2.

(10 points)

(3) Find the Maclaurin series for f(x)=cos2x directly, using the definition.

(10 points)

(4) Find the Maclaurin series for $x^4 e^{x^3}$ (5 points)

(There are easy ways and there are hard ways this can be done)

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(5) Find the Taylor series for $f(x)=1/x^2$ centered at $a=2$. (Assume that f has a power series expansion.

(10 points)

(6) Find the length of the curve $y=x^{2/3}$ from (1,1) to $(2\sqrt{2}, 2)$

(10 points)

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(7) Find the area of the surface generated by rotating the curve $y = 2\sqrt{x}$, $4 \leq x \leq 9$ about the x axis.
Set up two different integrals,
(a) by integrating with respect to x, and
(b) by integrating with respect to y.
Evaluate either one of these integrals.

(10 points)

(8) Using the geometric series for $\frac{1}{1-x}$ find a power series representation for $\frac{5x}{1+3x}$ and determine the radius of convergence.

(10 points)

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(9) Use series to compute $\int_0^{1/2} x^2 e^{-x^2}\,dx$ with |error| < 0.001. (10 points)

(10) (a) Approximate the function $f(x) = x \ln x$ by $T_3(x)$, the third degree Taylor Polynomial centered at a=1.

(b) Use Taylor's Inequality to estimate the accuracy of the approximation when x lies in the interval $0.9 \le x \le 1.1$

(c) Use $T_3(x)$ to approximate (1.01) ln(1.01)

(10 points)